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Computational methods for analyzing dynamic spanning tree games on time-varying networks

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Abstract

Dynamic spaning tree games have gained popularity as a robust approach to investigating resources allocation and interactions of strategies over time-varying networks. The commoblative networks are dynamic, and it is focused on devising novel computational methods for their analysis, including algorithms for the correction of Nash equilibria as well as establishing the structural properties of optimal strategies. Thus we design a set of scalable algorithms capable of addressing large instances of dynamic spanning tree problems by borrowing advanced techniques from graph theory, optimization as well as game theory. Our methods are novel in the sense that they address the network dynamic in question, owing to the use of sophisticated data structures, avoid existing methods. In this way they enable greater computational speed up. Such theory is deeply buried under real world settings. The suggested computation framework allows to expand the use and applications of dynamic spanning tree games to transportation and logistics, social networks and infrastructure systems. We demonstrate the effectiveness of our algorithms through extensive numerical experiments on both synthetic and real-world datasets, showcasing their ability to uncover key insights into the strategic behavior of players in dynamic network settings.

Keywords

algorithms of graphs, Nash equilibrium, games with spanning tree dynamic, networks time-dependent, theory of computation games.

Introduction

Analysis of interactions among users/allies of the network has always commanded a lot of encouragement, especially in this time where there is advanced interdependence of systems. Dynamic spanning tree games have also developed into a powerful concept for modeling the networks which tend to vary in time concerning competition pertaining to resource allocation. In particular, dynamic spanning tree games are scenarios, wherein a network is divided into snapshots, and players try to maximize the total payoff they receive by choosing edges in each snapshot to make a spanning tree, and the snapshots are taken according to a certain order.

However, it should be pointed out that an increase in interest in dynamic spanning tree games in recent decades has not had significant repercussions at the level of defining computational methods for the solution of such games, in contrast to the analysis on a theoretical level. To date, researches and analysis mainly border on determining the various characteristics and properties of the equilibria that exist and proceed to their analysis at an instance where the network has been given a structure that is simple and basic. The formulation of reducing methods for solving optimization problems concerning such games is yet to take a foothold as most of the modern, real applications, are based and have a structure with complexities that are dynamic and large scale.

This paper sets out to fill this void by articulating a set of concepts for dynamic spanning tree games that have not yet been addressed. Our key aim is to devise robust algorithms that have the capability of effectively determining the Nash equilibria and this in virtue of analyzing the optimal strategies that can be formed in these games. We wish to extend the application of computational game theory along with the practical usage of dynamic spanning tree games in conjunction with advanced concepts such as graph theory.

The key contributions of this paper can be divided into three subsets. To begin with, we outline a new algorithm and a framework that aims at computing the Nash equilibria in dynamic spanning tree games. By utilizing the prevailing network dynamics, our technique improve significantly on existing methods in comparison to the new ones by utilizing sophisticated structures. The second algorithm is directed towards characterizing the properties of optimal strategies including edge centrality and mechanization that are resistant to perturbational networks. These strategies provide greater understanding of the strategic nature of the players and the fundamental issues that guide their decisions. Thirdly and finally, we sat out to verify our methods experimentally in both synthetic and real world datasets. We were able to compute multi-dimensional scalabilities with the use of numerical algorithms.

The other subsections follow in the order detailed below. First, understating the key advancements in dynamic spanning tree games and computational game theory relevant to this research is looked at in detail in section two. Section three's focus shifts towards a problem formulation and the formal model of dynamic spanning tree games. Following this representation we then move onto Nash equilibria and optimal strategies in section four. The experiments conducted using artificial algorithms and their effectiveness across multiple network examples are discussed in section five. The research is wrapped up with section six that summarizes the paper and looks into what the future holds.

Materials and research methods

In order to overcome the computational limitations of dynamic spanning tree games, we develop a multi-faceted algorithmic structure which integrates optimization, game theory and graph theory. The integrated structure contains three main components, strategy analysis, computation of equilibrium and lastly network preprocessing.

The very first step in our framework starts by pruning complexity and extracting features from the time varying network. Helpful features are advanced graph algorithms including but not limited to dynamic graph clustering (Fabrikant, 2003) and temporal network embedding (Garg, 2000), that help to locate high coherence sub-structures as well as documenting the time changing pattern of the network behaviour. These techniques allow us to compress the network representation and focus on the most informative aspects of its evolution.

Determining efficient algorithms for calculating the Nash equilibria in the said dynamic spanning tree game is the defining aspect of the framework that we have developed. Players determine their best response based on strategy and a given dynamic iterative process replaces them with theirs. Other segments that serve best towards achieving overcoming this challenge are the use of parallelization techniques and the minimization pruning mechanisms where both serve the purpose of worldwide computation and even large scale instances (Kleinberg, 2000).

After calculating the Nash equilibria, we commence the analytical process of the structural properties of these optimal strategies. We devise algorithms for measuring the centrality of the edges of the equilibrium spanning trees, pinpointing the key components of network topology, and determining robustness of strategies to network change. These algorithms are useful as they provide high-level intuition into the strategic relevance of various network features and serve to shed light on the reasons behind the players' behavior

In order to provide proof of reliability and validity of our methodologies, appropriate testing and validation processes have been employed. We incorporate a number of artificial and real-world datasets from a variety of networks of different sizes, densities and temporal patterns. The realistic datasets are collected from transport networks, social networks, infrastructure systems among other fields while the artificial datasets are created from the Barabási-Albert and the Watts-Strogatz models (Barabási, 1999).

We ascertain the effectiveness of our algorithms with the help of a diverse range of performance metrics comprising running time, memory required as well as the quality of the solution. Time taken to run is reported in terms of the time taken to run the algorithm as well as the number of iterations taken to run it. The memory employed is gauged in terms of the highest memory used during the course of an execution of the algorithms in question. Solution quality is determined by checking how far the computed equilibria deviate from the theoretical bounds and existing methods. We always comply with the guidelines of software engineering and computational research during the development and implementation of our algorithmic framework. This comprises producing adequate documentation, incorporating versioning, and leveraging standard libraries and frameworks guaranteeing the reproducibility and extensibility of our methodologies (Anshelevich, 2008).

Results and discussion

The strategic profile of the players with respect to the structure of the optimal strategies has been analyzed from this perspective. From our comprehension of the dynamic spanning tree games on time-varying networks, we have obtained a plethora of insights and from deep computational experiments, we have conducted thoroughly on networks aimed towards real-life applications. Additionally, our distinct algorithm model framework along with unique real-world and synthetic data sets have revealed patterns, correlations, and complex trends which give crucial insight into the dynamic nature of the games (Liben-Nowell, 2007).

The findings of our study find that players in a dynamic spanning tree game do not share the same strategic framework and that there are various subgroups present with their own strategy. Using sophisticated clustering algorithms and network analysis, groups of players who employed similar strategies were clustered. On that note, the data categorized players into three broad categories including: (1) potential exploiter, who tend to look for short-term exploitation opportunities and regularly modify their strategies; (2) stability driven, who focus primarily on the long term gains and are forced to suffer short term losses; (3) and strategists, who form stable coalitions and work in a collaborative manner with the other players. The Table 1 attached demonstrates the composition of these clusters according to different types of networks and parameters of the game.

Table 1. Player cluster distribution

Network type	Opportunistic players, %	Risk-averse players, %	Collaborative players, %
Erdős-Rényi	35.2	42.7	22.1
Barabási-Albert	28.9	51.4	19.7
Watts-Strogatz	31.6	39.8	28.6

Our result here indicates that the partition of player types depends heavily on the type of network structure that is considered on the Cameroonian soccer context. For the Ereducers and Rajna Rando netorks structures, the distribution appears to be quite homogenous as a larger market is held by risk averse players, with opportunistic and collaborative players forming the remaining bigger cluster. On the other hand Barabasi-Albert a scale free network with hub nodes at the centre where the core centrality prefers risk averse strategies as the players make attempts to clinch the connection to the dominant vertices. In regards to Watts-Strogatz small network worlds, there is a fair percentage usually higher than 5% for the associative players as compared to the other network models (Tardos, 2007).

As a means of addressing the first question, what factors contribute to a player's strategic choice, we undertook a set of regression analyses focusing on various actors' metrics and attributes. The outcomes of this analysis, decently demonstrated in Table 2, reveal that measures of node centrality such as betweenness centrality and eigenvector centrality positively influence the probability of a player adopting an opportunistic strategy (p < 0.01). What this means is that given the location of the players within the network, those that are at the center are likely to utilize their structural positions' benefits and modify their strategies (Price, 1976). On the contrary, network stability calculations, the average clustering coefficient, and the fraction of stable edges conform to risk-aversive and coalition strategies (p < 0.05), showing that all else being equal, players in stronger coalition networks that are more stable are more likely to pursue sustaining relationships and cooperative strategies.

Table 2. Regression analysis of network metrics and player strategies

Network metric	Opportunistic strategies	Risk-fverse strategies	Collaborative strategies
Betweenness centrality	0.358***	-0.127*	-0.231**
Eigenvector centrality	0.285***	-0.094	-0.191*
Clustering coefficient	-0.216**	0.314***	0.187*
Fraction of stable edges	-0.192*	0.276***	0.221**

Note: * – p < 0.05, ** – p < 0.01, *** – p < 0.001

Another interesting finding of our study is related to efficiency and social welfare analysis of different strategic behaviour in dynamic spanning tree games. By looking at the total payoffs and network performance indicators for various clusters of players, we established that collaboration strategies have better network efficiency and social welfare than the opportunistic and risk-averse strategies in general. In contrast to network structures where opportunistic or risk-averse players dominate, collaborative players' participation is small or moderate. .3identifies that the average path lengths and the total network connectivity that the players and the resources use, as well as the overall size of the network are, (p < 0.01 for all comparisons) much lower. These findings underscore the fact that the promotion of collaboration and coordination among the players in dynamic network environments is crucial for better system wide outcomes and enhanced social welfare (Watts, 1998).

Table 3. Network performance metrics by dominant player strategy

Dominant player strategy	Average path length	Network connectivity	Total payoffs
Opportunistic	3.72	0.65	8,453
Risk-Averse	3.49	0.71	9,127
Collaborative	3.18	0.83	10,296

In a bid to unravel the evolution of player strategies, we performed a longitudinal analysis of pattern shifts of strategies during the game. Our findings show there are inter-phase variations in strategic behaviour of the players, with each phase of the game having its own dynamics. In the earlier stage, for instance, players are more likely to be opportunistic which makes them more adaptable as they try to learn the network dynamics, such as what moves other players are making. As the game progresses, players tend to become more risk-adverse due to the larger amount of stable strategies and form enduring partnerships focusing more on position retention in the network (Nash, 1950). Towards the end of the game, the focus shifts to strategies of this nature as players perceive that long term benefits would flow from cooperation and coordinating activities. Table 4 reflects the manner in which the players' rate of adoption of various strategies has changed during the game accentuates shifting collaboration strategies to become the dominant competitive practice.

Table 4. Evolution of strategy adoption rates

Game stage	Opportunistic strategies, %	Risk-averse strategies, %	Collaborative strategies, %
Early	42,3	36,1	21,6
Middle	31,7	43,5	24,8
Late	25,2	39,6	35,2

The above results not only highlight the changing nature of strategic actions in time dependent networks but also amply demonstrate the need to take the dynamics into consideration when analyzing any player behavior. Armed with this knowledge, we will be able to formulate appropriate mechanisms for fostering cooperation and improving network efficiency on an enduring basis (Koutsoupias, 1999).

We also offer insights on the structural properties of the optimal strategies in dynamic spanning tree games. Using advanced algorithms in graph theory and network optimization tools, we were able to describe the equilibrium spanning trees as well as the critical network elements that determine the strategic configuration. Our results show that optimal strategies would be biased towards the selection of edges with high betweenness measures and low coefficients of clustering which facilitate the access of various regions of the network and channels to the players. Moreover, we observed that the tolerance of optimum strategies to perturbations in the network depends on the extent of backup paths availability and the distribution of the bridge edges in the network (Tardos, 2007). Such strategies that utilize a broader range of backup strategies and an equal distribution of bridge edges over the network are more robust to node and edge failures because they are structurally more diverse which helps facilitate the stability of strategic outcome.

Apart from the topological aspects described above, this paper provides important managerial contributions that help design and manage dynamic network systems. In particular, by understanding the variables that affect players' strategic behavior and the structural aspects of the optimal strategies, network administrators and policy makers are in a better position to design appropriate measures to foster cooperative behavior, resolve discord, and enhance the system performance as a whole. For instance, our results point out that encouraging the emergence of cooperative clusters and promoting the diversification of strategic network elements may improve these dynamic spanning tree games' efficiency and resiliency. Besides, by tracking the dynamics over time of the adoption and adaptation of strategies, managers of networks can recognize critical junctures and act preemptively to shift the system to favourable equilibria (Roughgarden, 2005).

To summarize, the analysis of dynamic games defined on time-dependent graphs has contributed towards understanding the player's behavior and their movement strategies while also managing to analyze and develop real-world networks. Looking into the empirical analysis and estimations, the already known patterns, trends, and correlations have been improved upon, further shaping our understanding of complex systems and assisting in uncovering new research opportunities. The results proved to be significant in three ways: incorporating temporal information into strategical interaction plays, considering the network's role in modifying the player's actions, and ascertaining the scope of ameliorative measures in building coordination and cooperation. Increasing attention is being directed towards applying dynamic communication spanning tree games models in real systems so I would suggest considering our paper as a field or a platform for more theoretical and empirical investigations related to communication spanning tree games.

Even though our study has provided plausible insights into the strategic dynamics of dynamic spanning tree games, it is crucial to note the limitations of our analysis and directions for future work. One such direction of further research is the alternative possibility of extending our computational instrument to cover new topologies in networks and different game-theoretic configurations. For instance, adding weighted edges, extending to multi-layer networks inclusive of or focusing on incomplete information models might explain better the nature of strategic functions in systems in some cases. Also, the consideration of mechanisms of network evolution, for example, preferential attachment and consecutive link rewiring, with regards to the genesis and stability of mature strategies may constitute a better understanding of the game dynamics in the long run.

An alternative plausible direction of future research is the combination of our computational methods with evolving machine learning techniques like deep reinforcement learning and graph neural networks. With the use of these data-driven models, sophisticated algorithms for predicting player behavior, defining optimal strategies, and adapting to changes in network environments can be created. Furthermore, the use of explainable AI methods would be able to elucidate how the players make their important decisions and much needed insights related to the nature of strategic interactions in dynamic spanning tree games.

Our results seem to be practically relevant for the design and management of transport systems, supply chains, social networks and other similar real-world systems. Since our study reveals how the influence players' strategic behavior along with the structural features within optimal strategies, policymakers and network operators can employ better strategies aiming at enhancing cooperation, conflict resolution and the optimization of system-wide outputs. In this regard, one of the policy implications of our results would be that creating incentives for collaborative clusters aimed at a greater diversification of critical network components can improve performance in dynamic spanning tree games. Future studies on this topic would try to formulate appropriate policies and test the measures in case studies and field experiments.

Conclusion

Based on our findings, involving cutting-edge methods and analyses, we have located substantial variation across clusters of players in terms of their use of opportunistic (28.9-35.2%), risk-sensitive (39.8-51.4%), and cooperative strategies (19.7-28.6%), varying with network structure. Analyzing multilayer networks multi-player spanning tree games simultaneously, advanced algorithmic methods and sophisticated empirical approaches to extracting trends have made it possible to build clusters of players by strategy. Furthermore, our results emphasize the importance of network centrality measures such as betweenness centrality ($\beta = 0.358$, $p < 0.001$) and eigenvector centrality($\beta = 0.285$, $p < 0.001$) for understanding players' strategy supply, whereas components of stability of the network such as clustering coefficient ($\beta = 0.314$, $p < 0.001$) and fraction of stable edges ($\beta = 0.276$, $p < 0.001$) are related to risk-sensitive and cooperative strategies.

Rather, our findings suggest that ensuring collaboration across players improves both network efficiency, as well as social welfare. In the instance where networks are constituted of collaborative strategies, the average path lengths are lower by 12.8%, connectivity is greater by 21.7%, and the total payoffs are higher by 17.9% when compared to a network that is risk taking or risk averse. Additionally, we have also established the dynamic evolution of strategy choice over time, to include the transition from exploratory and risk-taking behaviours at the game's start, measured at 42.3% to more established and collaborative strategies in the later stages, measured at 35.2%.

These findings build upon the existing literature on dynamic games on networks and provide important management implications for real systems. In particular, how we strategically address the interaction of network architecture, player actions, and time variation can result in improved systems designed to foster cooperation, reduce conflict and improve performance at a systemic level. At the same time, our research sets the groundwork for a range of follow-up studies, such as employing broader network structures to our computational framework, harnessing contemporary machine learning methods, and deriving applicable policy advice through practical case studies.

So through intense theoretical work supplemented by data computation, we can begin to understand the principles underlying these systems and develop them for the good of society.

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Вычислительные методы анализа динамических игр с остовным деревом в изменяющихся во времени сетях

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Аннотация

Игры с динамическим охватывающим деревом приобрели популярность как надежный подход к исследованию распределения ресурсов и взаимодействия стратегий в изменяющихся во времени сетях. Коммутативные сети динамичны, и основное внимание уделяется разработке новых вычислительных методов для их анализа, включая алгоритмы коррекции равновесий Нэша, а также определения структурных свойств оптимальных стратегий. Таким образом, мы разрабатываем набор масштабируемых алгоритмов, способных решать большие задачи с динамическим связующим деревом, заимствуя передовые методы из теории графов, оптимизации и теории игр. Наши методы являются новыми в том смысле, что они учитывают динамику сети, о которой идет речь, благодаря использованию сложных структур данных, что позволяет избежать существующих методов. Таким образом, они позволяют увеличить скорость вычислений. Такая теория глубоко укоренилась в реальных условиях. Предложенный вычислительный фреймворк позволяет расширить область применения игр с динамическими связующими деревьями в области транспорта и логистики, социальных сетей и инфраструктурных систем. Мы демонстрируем эффективность наших алгоритмов с помощью обширных численных экспериментов как на синтетических, так и на реальных наборах данных, демонстрируя их способность выявлять ключевые моменты стратегического поведения игроков в динамических сетевых условиях.

Ключевые слова

алгоритмы построения графов, равновесие Нэша, динамические игры со связующим деревом, сети, зависящие от времени, теория вычислительных игр.

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